

Insurance Operations --- Note on Mortality Tables and Life Insurance Mathematics

The development of life insurance was preceded by the creation of a reliable mortality table along with the mathematics to calculate probabilities of death. [see: <http://www.aaimedicine.org/journal-of-insurance-medicine/jim/1988/020-04-0012.pdf>] The early beginnings of actuarial mathematics start with the French mathematician – Blaise Pascal who in the 17th century developed methods for determining the probability of events occurring in games of chance. [e.g. the sum of the probabilities on a set of events must add to 1, for independent events the probability of A and B = prob(A) * prob(b), for non-mutually exclusive events – the prob(A or B) = prob (A) + prob(B) – prob (A and B)]. As a renaissance scholar Pascal was responsible for creation of the first adding machine, results in fluid mechanics leading to development of the weather barometer, theoretical findings in the area of projective geometry, as well as, papers on how mathematical methods may be used to support the foundations of Christian faith using probabilistic arguments [see:

<http://www.ideafinder.com/history/inventors/pascal.htm>]. It is remarkable that with all of Pascal's discoveries he died so young, at the age of 39.

The development of a mortality table was difficult because the collection of data involved obtaining statistics on the probability of death at each individual age. Under such circumstances there are two ways to create a mortality table: (1) **Closed Group Model** (2) **Open Group Model**

The **closed group model** involves taking a large group of people at age 0 and recording deaths from that group from age 0 to 100 [the terminal age of the mortality table]. The closed group method is a time series type of statistical technique. The disadvantage of this method is that it would take 100 years to fully develop the table and then it would likely be obsolete due to changes in mortality due to medical improvements. On the other hand, the closed group method is used in medicine to determine drug efficacy or the onslaught of disease for a particular population. The advantage of the closed group model is to be able to see how a particular risk might be impacting a separate population during a short time frame.

For life insurance, the **open group model** is used to create a mortality table using cross section time series data that can be updated over short periods of time. The open group model assumes that the probabilities of death remain fixed at each age allowing a table to be created using the probability of death for each age [${}_1q_x$ = probability a person age x dies before reaching age x+1 defined to be :

$${}_1q_x = \frac{l_x - l_{x+1}}{l_x}, \text{ where } l_x - l_{x+1} = d_0$$

l_x

The open group mortality method starts with doing a study of individuals living at each age and then recording the number of deaths during the next year. By taking the number of recorded deaths at each age [$l_x - l_{x+1} = d_x$, those who die between ages x and $x+1$] and then dividing by the number of individuals who start out at age x [l_x] you have the probability a person age x dies before attaining age $x+1$ or ${}_1q_x$. Once the q 's have been calculated you can develop the mortality table along the following lines: (1) select an initial number for those age 0, [l_0] this is sometimes called the radix to a mortality table, so for example suppose you choose the radix to be 1,000,000 (2) next use the probability of death for the first year, ${}_1q_0$ to determine the number of people dying in the first year [d_0] using the following relationship:

$${}_1q_0 = \frac{d_0}{l_0} \quad \text{which provides} \quad d_0 = {}_1q_0 \times l_0$$

To illustrate, suppose that you have done a mortality study from ages 0 to 5 and have calculated q 's at each of these ages and found the following results:

<u>Age(x)</u>	<u>l_x</u>	<u>d_x</u>	<u>${}_1q_x$</u>
0			.00708
1			.00176
2			.00152
3			.00146
4			.00140
5			.00135

If you select an initial radix of 1,000,000 the first line of this mortality table would be as follows:

<u>Age(x)</u>	<u>l_x</u>	<u>d_x</u>	<u>${}_1q_x$</u>
0	1,000,000	70800	.00708

where $d_0 = 1,000,000 \times .00708$

The next line for age 1 in the mortality table would be found based on the following relationship:

$$l_{x+1} = l_x - d_x \quad \text{which may be translated as follows :}$$

l_{x+1} the number of people living to age $x+1$ represent those who are living one year earlier at age x [l_x] less those who don't make it to age $x+1$ due to death [d_x].

So, $l_1 = l_0 - d_0$ and in this case that would be:

$l_1 = 1,000,000 - 70,800 = 9,929,200$ and the next line to the mortality table would be:

<u>Age(x)</u>	<u>l_x</u>	<u>d_x</u>	<u>${}_1q_x$</u>
0	1,000,000	70800	.00708
1	9,929,200	17475	.00176

where $d_1 = 9,929,200 \times .00176 = 17,475$

In similar fashion, the next line for age 2 would be:

$$l_2 = 9,929,200 - 17,475 = 9,911,725$$

and $d_2 = 9,911,725 \times .00152 = 15,066$ to render the following mortality table for the first three ages:

<u>Age(x)</u>	<u>l_x</u>	<u>d_x</u>	<u>${}_1q_x$</u>
0	1,000,000	70800	.00708
1	9,929,200	17475	.00176
2	9,911,725	15066	.00152

As an extra credit assignment complete the rest of this mortality table for ages 3-5.

<u>Age(x)</u>	<u>l_x</u>	<u>d_x</u>	<u>${}_1q_x$</u>
0	1,000,000	70800	.00708
1	9,929,200	17475	.00176
2	9,911,725	15066	.00152
3			.00146
4			.00140
5			.00135

Once you have developed a mortality table you can use these figures to determine probabilities of life and death at various ages. Using a portion of the 1958 CSO Mortality Table below, calculate the following: (a) prob[a person age 20 lives 5 years to age 25] and

(b) prob[a person age 20 lives to age 25 and then dies in the next year before reaching age 26].

Age(x)	l_x	d_x	q_x
19	9,681,840	16,846	0.0017400
20	9,664,994	17,300	0.0017900
21	9,647,694	17,655	0.0018300
22	9,630,039	17,912	0.0018600
23	9,612,127	18,167	0.0018900
24	9,593,960	18,324	0.0019100
25	9,575,636	18,481	0.0019300
26	9,557,155	18,732	0.0019600
27	9,538,423	18,981	0.0019900
28	9,519,442	19,324	0.0020300
29	9,500,118	19,760	0.0020800
30	9,480,358	20,193	0.0021300
31	9,460,165	20,718	0.0021900
32	9,439,447	21,239	0.0022500
33	9,418,208	21,850	0.0023200
34	9,396,358	22,551	0.0024000
35	9,373,807	23,528	0.0025100